Solutions, Problem Set 8

1. VIA THE F.T.V.S., WE CAN TRANSLATE ANY COMPUTATIONAL QUESTION IN A FINITE-DIMENSIONAL VECTOR SPACE V INTO A PROBLEM ABOUT COLUMN VECTORS; ALL THAT WE NEED IS TO FIND A $\underline{\text{BASIS}}$ $\mathcal{B}$ FOR THE VECTOR SPACE IN QUESTION AND USE THE ISOMORPHISMS $[\mathcal{B}]$ AND $[\mathcal{B}]^{-1}$ — WHEN FINISHED, WE SHOULD BE SURE TO TRANSLATE OUR SOLUTION BACK TO THE CONTEXT OF THE ORIGINAL VECTOR SPACE.

2. LET $\mathcal{C} = \{1+x, x+x^2, 1+x^2\} \subset P_2(x)$. TAKING THE USUAL BASIS $(1, x, x^2)$ FOR $P_2(x)$, THE COLLECTION $\mathcal{C}$ CORRESPONDS TO THE COLLECTION $\mathcal{C}' = \left\{ \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix} \right\} \subset \mathbb{R}^3$.

TO EXAMINE $\mathcal{C} \subset P_2(x)$, JUST LOOK AT $\mathcal{C}' \subset \mathbb{R}^3$:

(a) $\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix} \rightsquigarrow \begin{bmatrix}\textcircled{1}&0&0\\0&\textcircled{1}&0\\0&0&\textcircled{1}\end{bmatrix}$ PIVOT IN EVERY ROW + COLUMN
$\therefore$ $\mathcal{C}'$ IS $\underline{\text{L.I.}}$ IN $\mathbb{R}^3$ AND $\underline{\text{SPANS}}$ $\mathbb{R}^3$,
SO IT IS A $\underline{\text{BASIS FOR } \mathbb{R}^3}$.

THUS, $\mathcal{C}$ IS L.I. IN $P_2(x)$, SPANS $P_2(x)$, AND IS A BASIS FOR $P_2(x)$.

(b) THE PROBLEM OF WRITING x^2 AS A L.C. OF $\mathcal{C}$ IS EQUIVALENT TO THAT OF WRITING $\begin{bmatrix}0\\0\\1\end{bmatrix}$ AS A L.C. OF $\mathcal{C}'$:

$$\begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \left[\begin{array}{ccc|c}1&0&1&0\\1&1&0&0\\0&1&1&1\end{array}\right] \end{array} \rightsquigarrow \left[\begin{array}{ccc|c}\textcircled{1}&0&0&-1/2\\0&\textcircled{1}&0&1/2\\0&0&\textcircled{1}&1/2\end{array}\right]$$

$$\therefore \begin{bmatrix}0\\0\\1\end{bmatrix} = -\tfrac{1}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} + \tfrac{1}{2}\begin{bmatrix}0\\1\\1\end{bmatrix} + \tfrac{1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

SO $\underline{x^2 = -\tfrac{1}{2}(1+x) + \tfrac{1}{2}(x+x^2) + \tfrac{1}{2}(1+x^2)}$.

(c) BECAUSE $\mathcal{C}$ IS A BASIS FOR $P_2(x)$ (SEE PART (a)), $\underline{\text{EVERY}}$ POLYNOMIAL IN $P_2(x)$ CAN BE WRITTEN AS A L.C. OF $\mathcal{C}$ IN $\underline{\text{EXACTLY ONE WAY}}$.

3. LET $\mathcal{C} = \{1-x, x-x^2, 1-x^2\} \subset P_2(x)$. TAKING THE USUAL BASIS $(1, x, x^2)$ FOR $P_2(x)$, $\mathcal{C}$ CORRESPONDS TO THE COLLECTION $\mathcal{C}' = \left\{ \begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix} \right\} \subset \mathbb{R}^3$.

TO EXAMINE $\mathcal{C} \subset P_2(x)$, JUST LOOK AT $\mathcal{C}' \subset \mathbb{R}^3$:

(a) WRITING $1-2x+x^2$ AS A L.C. OF $\mathcal{C}$ IS EQUIVALENT TO WRITING $\begin{bmatrix}1\\-2\\1\end{bmatrix}$ AS A L.C. OF $\mathcal{C}'$:

$$\begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \left[\begin{array}{ccc|c}1&0&1&1\\-1&1&0&-2\\0&-1&-1&1\end{array}\right] \end{array} \rightsquigarrow \begin{array}{c} \begin{array}{ccc} &&\text{FREE} \end{array} \\ \left[\begin{array}{ccc|c}\textcircled{1}&0&1&1\\0&\textcircled{1}&1&-1\\0&0&0&0\end{array}\right] \end{array}$$

$\underline{\text{CONSISTENT}}$, $\therefore$ $\begin{bmatrix}1\\-2\\1\end{bmatrix}$ $\underline{\text{CAN}}$ BE WRITTEN AS A L.C. OF $\mathcal{C}'$

$\therefore$ $1-2x+x^2$ $\underline{\text{CAN BE WRITTEN}}$ AS A L.C. OF $\mathcal{C}$.

(b) AS ABOVE, BUT FOR $1-x+x^2 \leftrightarrow \begin{bmatrix}1\\-1\\1\end{bmatrix}$:

$$\begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \left[\begin{array}{ccc|c}1&0&1&1\\-1&1&0&-1\\0&-1&-1&1\end{array}\right] \end{array} \rightsquigarrow \begin{array}{c} \begin{array}{ccc} &&\text{FREE} \end{array} \\ \left[\begin{array}{ccc|c}\textcircled{1}&0&1&1\\0&\textcircled{1}&1&0\\0&0&0&1\end{array}\right] \end{array} \leftarrow !$$

IN$\underline{\text{CONSISTENT}}$, $\therefore$ $\begin{bmatrix}1\\-1\\1\end{bmatrix}$ $\underline{\text{CANNOT}}$ BE WRITTEN AS A L.C. OF $\mathcal{C}'$

$\therefore$ $1-x+x^2$ $\underline{\text{CANNOT BE WRITTEN}}$ AS A L.C. OF $\mathcal{C}$.

(c) LOOKING AT OUR MATRIX REDUCTIONS ABOVE, WE SEE THAT (ELIMINATING THE THIRD VECTOR),

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\} \text{ IS A BASIS FOR SPAN } \mathcal{C}'$$

$$\therefore \underbrace{\{1-x,\ x-x^2\}}_{} \text{ IS A BASIS FOR SPAN } \mathcal{C}.$$

* IN TERMS OF SPAN $\mathcal{C}$, PART (a) SAYS THAT $1-2x+x^2 \in \text{SPAN } \mathcal{C}$ AND PART (b) SAYS THAT $1-x+x^2 \notin \text{SPAN } \mathcal{C}$.

(d) JUST AS IN PARTS (a) AND (b), BUT FOR AN UNKNOWN POLYNOMIAL $a+bx+cx^2 \longleftrightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix}$:

$$\begin{array}{r} \\ +1 \curvearrowright \\ \text{THEN } +1 \curvearrowright \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & a \\ -1 & 1 & 0 & b \\ 0 & -1 & -1 & c \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} & & \text{FREE} & \\ \enclose{circle}{1} & 0 & 1 & a \\ 0 & \enclose{circle}{1} & 1 & a+b \\ 0 & 0 & 0 & a+b+c \end{array}\right]$$

IN ORDER FOR THIS SYSTEM TO BE CONSISTENT, WE MUST HAVE $a+b+c=0$ (AND, CONVERSELY, IF $a+b+c=0$, THE SYSTEM WILL BE CONSISTENT).

$$\therefore \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \text{SPAN } \mathcal{C}' \iff a+b+c=0$$

$$\text{SO } a+bx+cx^2 \in \text{SPAN } \mathcal{C} \iff \underline{a+b+c=0}$$

4. $[\mathcal{C} = \{1+t+t^3,\ t+t^2-t^3,\ 1-t^2\} \subset P_3(t)]$

TAKING THE USUAL BASIS $(1, t, t^2, t^3)$ FOR $P_3(t)$,

$$\mathcal{C} \text{ CORRESPONDS TO } \mathcal{C}' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \right\} \subset \mathbb{R}^4.$$

$$\text{(a)} \quad \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} \enclose{circle}{1} & 0 & 0 \\ 0 & \enclose{circle}{1} & 0 \\ 0 & 0 & \enclose{circle}{1} \\ 0 & 0 & 0 \end{bmatrix}$$

PIVOT IN EVERY COLUMN, SO $\mathcal{C}'$ IS L.I. IN $\mathbb{R}^4$

$\therefore \mathcal{C}$ IS L.I. IN $P_3(t)$.

(b) EXTEND $\mathcal{C}'$ TO A BASIS FOR $\mathbb{R}^4$:

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} \enclose{circle}{1} & 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & \enclose{circle}{1} & 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & \enclose{circle}{1} & 0 & 1/2 & -1 & -1/2 \\ 0 & 0 & 0 & \enclose{circle}{1} & -1 & 1 & 0 \end{bmatrix}$$

$$\therefore \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\} \text{ IS A BASIS FOR } \mathbb{R}^4$$

$$\text{SO } \underbrace{\{1+t+t^3,\ t+t^2-t^3,\ 1-t^2,\ 1\}}_{} \text{ IS A BASIS FOR } P_3(t).$$

5. $\left[\mathcal{C} = \{1-2z+3z^2,\ 13z^2+6z-1,\ 23z^2+14z-3,\ 2-12z-23z^2\} \subset P_2(z) \right]$

TAKING THE USUAL BASIS $(1, z, z^2)$ FOR $P_2(z)$,

$\mathcal{C}$ CORRESPONDS TO THE COLLECTION $\mathcal{C}' = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \underbrace{\begin{bmatrix} -1 \\ 6 \\ 13 \end{bmatrix}, \begin{bmatrix} -3 \\ 14 \\ 23 \end{bmatrix}}, \begin{bmatrix} 2 \\ -12 \\ -23 \end{bmatrix} \right\} \subset \mathbb{R}^3.$

↳ BE CAREFUL WITH THE ORDER OF THE ENTRIES!

(a) $$\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \end{array}$$
$$\begin{bmatrix} 1 & -1 & -3 & 2 \\ -2 & 6 & 14 & -12 \\ 3 & 13 & 23 & -23 \end{bmatrix} \rightsquigarrow \begin{array}{cccc} & & \text{FREE} & \\ \boxed{1} & 0 & -1 & 0 \\ 0 & \boxed{1} & 2 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{array} \rightarrow \begin{array}{l} a_3 : \text{FREE} \\ a_1 = a_3 \\ a_2 = -2a_3 \\ a_4 = 0 \end{array}$$

THERE IS NOT A PIVOT IN EVERY COLUMN, SO $\mathcal{C}'$ IS NOT L.I. IN $\mathbb{R}^3$. FINDING A NONTRIVIAL L.R. ON $\mathcal{C}'$ AMOUNTS TO FINDING A NONTRIVIAL SOLUTION; TAKE THE FREE VARIABLE a_3 TO BE 1, GIVING $a_1 = 1$, $a_2 = -2$, $a_4 = 0$.

$$\text{THUS, } 1\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} - 2\begin{bmatrix} -1 \\ 6 \\ 13 \end{bmatrix} + 1\begin{bmatrix} -3 \\ 14 \\ 23 \end{bmatrix} + 0\begin{bmatrix} 2 \\ -12 \\ -23 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $\mathcal{C}$ IS LINEARLY DEPENDENT AND

$$1(1-2z+3z^2) - 2(13z^2+6z-1) + 1(23z^2+14z-3) + 0(2-12z-23z^2) = 0$$

(b) CONSIDERING OUR MATRIX REDUCTION ABOVE, WE SEE THAT $\mathcal{C}'$ ALREADY SPANS $\mathbb{R}^3$, AND THAT ELIMINATING THE THIRD VECTOR WOULD REDUCE IT TO A BASIS FOR $\mathbb{R}^3$. DOING THE SAME WITH $\mathcal{C}$, WE OBTAIN THAT BY ISOMORPHISM, $\{1-2z+3z^2,\ 13z^2+6z-1,\ 2-12z-23z^2\}$ IS A BASIS FOR $P_2(z)$.

6. THESE FUNDS' VALUES CAN BE SCALED AND ADDED, SO WE'RE DEALING WITH VECTORS! THEY HAVE THREE RISK COMPONENTS, WHICH IF WE READ CORRECTLY GIVE A BASIS:

FUND A = 20$\underline{\text{% HIGH-RISK}}$ + 50$\underline{\text{% MEDIUM-RISK}}$ + 30$\underline{\text{% LOW-RISK}}$

ETC. — OUR BASIS (ODD AS IT LOOKS) COULD BE

$\mathcal{B}$ = (% HIGH-RISK, % MEDIUM-RISK, % LOW-RISK)

THEN OUR FUNDS' COORDINATES ARE $\begin{bmatrix} 20 \\ 50 \\ 30 \end{bmatrix}, \begin{bmatrix} 20 \\ 10 \\ 70 \end{bmatrix}$, AND $\begin{bmatrix} 50 \\ 40 \\ 10 \end{bmatrix}$.

NOW, TO BALANCE RISK, WE WANT

$$a_1\begin{bmatrix} 20 \\ 50 \\ 30 \end{bmatrix} + a_2\begin{bmatrix} 20 \\ 10 \\ 70 \end{bmatrix} + a_3\begin{bmatrix} 50 \\ 40 \\ 10 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}$$

← SINCE WE'RE ONLY INTERESTED IN RELATIVE PROPORTIONS, WE COULD TAKE ANY OTHER EQUAL VALUES HERE... BUT 100 PUTS THEM ON A NICE SCALE WITH THE OTHER NUMBERS.

SOLVING:
$$\left[\begin{array}{ccc|c} a_1 & a_2 & a_3 & \\ 20 & 20 & 50 & 100 \\ 50 & 10 & 40 & 100 \\ 30 & 70 & 10 & 100 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 3/4 \\ 0 & \boxed{1} & 0 & 11/12 \\ 0 & 0 & \boxed{1} & 4/3 \end{array}\right]$$

$\therefore a_1 = 3/4,\ a_2 = 11/12,\ a_3 = 4/3,$

SO WE SHOULD BUY FUNDS A, B, AND C

IN PROPORTIONS $\frac{3}{4} : \frac{11}{12} : \frac{4}{3}$, OR MORE SIMPLY, $\underline{9 : 11 : 16}$.

7. BY TAKING BASES CONSISTING OF THE RELEVANT ELEMENTS (IN UNITS OF ATOMS), WE CAN TRANSFORM ANY CHEMICAL EQUATION INTO A PROBLEM OF L.C.'S OF COLUMN VECTORS; IF WE MOVE ALL VECTORS TO THE LEFT, WE FIND THAT A BALANCED CHEMICAL EQUATION CORRESPONDS TO A NONTRIVIAL LINEAR RELATION ON THE VECTORS REPRESENTING THE COMPOUNDS INVOLVED! ☺

(a) TAKING THE BASIS (Al, H, O, S), WE FIND THAT

$$Al(OH)_3 \leftrightarrow \begin{bmatrix}1\\3\\3\\0\end{bmatrix}, \quad H_2SO_4 \leftrightarrow \begin{bmatrix}0\\2\\4\\1\end{bmatrix}, \quad Al_2(SO_4)_3 \leftrightarrow \begin{bmatrix}2\\0\\12\\3\end{bmatrix}, \quad H_2O \leftrightarrow \begin{bmatrix}0\\2\\1\\0\end{bmatrix}$$

SO WE'RE SOLVING

$$\begin{array}{cccc} AH & SA & AS & W \\ \hline 1 & 0 & 2 & 0 \\ 3 & 2 & 0 & 2 \\ 3 & 4 & 12 & 1 \\ 0 & 1 & 3 & 0 \end{array} \rightsquigarrow \begin{array}{cccc} & & & \text{FREE} \\ \hline \textcircled{1} & 0 & 0 & 1/3 \\ 0 & \textcircled{1} & 0 & 1/2 \\ 0 & 0 & \textcircled{1} & -1/6 \\ 0 & 0 & 0 & 0 \end{array}$$

W: FREE
$AH = -\frac{1}{3}W$
$SA = -\frac{1}{2}W$
$AS = \frac{1}{6}W$

TO GET INTEGER SOLUTIONS, TAKE $W=6$: $AH=-2$, $SA=-3$, $AS=1$

$$\therefore \; -2\begin{bmatrix}1\\3\\3\\0\end{bmatrix} - 3\begin{bmatrix}0\\2\\4\\1\end{bmatrix} + 1\begin{bmatrix}2\\0\\12\\3\end{bmatrix} + 6\begin{bmatrix}0\\2\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\text{So } 2\begin{bmatrix}1\\3\\3\\0\end{bmatrix} + 3\begin{bmatrix}0\\2\\4\\1\end{bmatrix} = 1\begin{bmatrix}2\\0\\12\\3\end{bmatrix} + 6\begin{bmatrix}0\\2\\1\\0\end{bmatrix}$$

SLIDE NEGATIVES OVER TO KEEP THE COEFFICIENTS POSITIVE — NOTE THAT THE ALGEBRA TELLS US WHAT GOES ON WHAT SIDE!

TRANSLATING BACK GIVES US THE BALANCED EQUATION:

$$2Al(OH)_3 + 3H_2SO_4 \longrightarrow Al_2(SO_4)_3 + 6H_2O.$$

(b) TAKING THE BASIS (C, H, O), WE HAVE:

$$C_2H_5OH \leftrightarrow \begin{bmatrix}2\\6\\1\end{bmatrix}, \quad O_2 \leftrightarrow \begin{bmatrix}0\\0\\2\end{bmatrix}, \quad CO_2 \leftrightarrow \begin{bmatrix}1\\0\\2\end{bmatrix}, \text{ AND } H_2O \leftrightarrow \begin{bmatrix}0\\2\\1\end{bmatrix},$$

GIVING THE SYSTEM

$$\begin{array}{cccc} E & OX & CD & W \\ \hline 2 & 0 & 1 & 0 \\ 6 & 0 & 0 & 2 \\ 1 & 2 & 2 & 1 \end{array} \rightsquigarrow \begin{array}{cccc} & & & \text{FREE} \\ \hline \textcircled{1} & 0 & 0 & 1/3 \\ 0 & \textcircled{1} & 0 & 1 \\ 0 & 0 & \textcircled{1} & -2/3 \end{array}$$

W: FREE
$E = -\frac{1}{3}W$
$OX = -W$
$CD = 2/3\,W$

TAKING $W=3$ FOR INTEGER SOLUTIONS, WE HAVE $E=-1$, $OX=-3$, $CD=2$, $W=3$, SO THE CHEMICAL EQUATION BALANCES AS

$$C_2H_5OH + 3O_2 \longrightarrow 2CO_2 + 3H_2O.$$

(c) TAKING THE BASIS (Cl, H, K, O), WE HAVE

$$KClO_3 \leftrightarrow \begin{bmatrix}1\\0\\1\\3\end{bmatrix}, \quad HCl \leftrightarrow \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \quad KCl \leftrightarrow \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \quad H_2O \leftrightarrow \begin{bmatrix}0\\2\\0\\1\end{bmatrix}, \quad Cl_2 \leftrightarrow \begin{bmatrix}2\\0\\0\\0\end{bmatrix}, \quad ClO_2 \leftrightarrow \begin{bmatrix}1\\0\\0\\2\end{bmatrix},$$

GIVING THE SYSTEM

$$\begin{array}{c} \begin{array}{cccccc} KA & HC & KI & W & Cl & CD \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

$$\rightsquigarrow \begin{array}{c} \begin{array}{cccccc} & & & & \text{FREE} & \text{FREE} \end{array} \\ \begin{bmatrix} \textcircled{1} & 0 & 0 & 0 & 1/3 & 5/6 \\ 0 & \textcircled{1} & 0 & 0 & 2 & 1 \\ 0 & 0 & \textcircled{1} & 0 & -1/3 & -5/6 \\ 0 & 0 & 0 & \textcircled{1} & -1 & -1/2 \end{bmatrix} \end{array}$$

Cl, CD: FREE

$KA = -\frac{1}{3}Cl - \frac{5}{6}CD$

$HC = -2Cl - CD$

$KI = \frac{1}{3}Cl + \frac{5}{6}CD$

$W = Cl + \frac{1}{2}CD$

* TWO FREE VARIABLES?!? — ONE WOULD JUST ACCOUNT FOR SCALING THE EQUATION, BUT TWO MEANS THAT MORE THAN ONE REACTION IS POSSIBLE (AT LEAST MATHEMATICALLY)... WE CAN FIND TWO SUCH BY SETTING ONE FREE VARIABLE TO ZERO AND THE OTHER TO SOMETHING SENSIBLE FOR INTEGER SOLUTIONS, E.G.:

① $\underline{Cl = 3, CD = 0}$ GIVES $KA = -1, HC = -6, KI = 1, W = 3$, I.E.,

$$\underline{KClO_3 + 6HCl \longrightarrow KCl + 3H_2O + 3Cl_2.}$$

OR ② $\underline{Cl = 0, CD = 6}$ GIVES $KA = -5, HC = -6, KI = 5, W = 3$, I.E.,

$$\underline{5KClO_3 + 6HCl \longrightarrow 5KCl + 3H_2O + 6ClO_2.}$$

AS WELL AS A WHOLE LATTICE OF OTHER [MATHEMATICAL] POSSIBILITIES!

- IN ORDER TO ALLOW ANY CHEMICAL REACTION TO BE EXPRESSED, WE COULD SIMPLY TAKE A BASIS CONSISTING OF ALL KNOWN ELEMENTS — GIVING A VECTOR SPACE OF DIMENSION 118 OR SO... ALL OF THE ABOVE PARTS TAKE PLACE WITHIN SOME SMALLISH SUBSPACES OF THIS UNIVERSAL MOLECULAR SPACE!