Solutions, Problem Set 16

1. (a) [$L : V \to W$ linear transformation]

$$\begin{tikzcd}
\mathbb{R}^n \arrow[rr, "{_{\mathcal{W}'}[L]_{\mathcal{V}'}}"] \arrow[d, shift left, "{[\mathcal{V}']}"] \arrow[dd, bend right=60, "{[\mathcal{V}]^{-1}[\mathcal{V}'] : \mathbb{R}^n \to \mathbb{R}^n}"'] & & \mathbb{R}^m \arrow[d, shift left, "{[\mathcal{W}']}"] \\
V \arrow[rr, "L"] \arrow[u, shift left, "{[\mathcal{V}']^{-1}}"] \arrow[d, shift left, "{[\mathcal{V}]}"] & & W \arrow[u, shift left, "{[\mathcal{W}']^{-1}}"] \arrow[d, shift left, "{[\mathcal{W}]}"] \\
\mathbb{R}^n \arrow[rr, "{_{\mathcal{W}}[L]_{\mathcal{V}}}"'] \arrow[u, shift left, "{[\mathcal{V}]^{-1}}"] & & \mathbb{R}^m \arrow[u, shift left, "{[\mathcal{W}]^{-1}}"] \arrow[uu, bend right=60, "{[\mathcal{W}']^{-1}[\mathcal{W}] : \mathbb{R}^m \to \mathbb{R}^m}"']
\end{tikzcd}$$

- $_{\mathcal{W}'}[L]_{\mathcal{V}'} = [\mathcal{W}']^{-1} L [\mathcal{V}']$
  $= [\mathcal{W}']^{-1}\left([\mathcal{W}]\, _{\mathcal{W}}[L]_{\mathcal{V}} [\mathcal{V}]^{-1}\right)[\mathcal{V}']$
  $= \left([\mathcal{W}']^{-1}[\mathcal{W}]\right) {_{\mathcal{W}}[L]_{\mathcal{V}}} \left([\mathcal{V}]^{-1}[\mathcal{V}']\right)$

AND, CONVERSELY,

CHANGE-OF-BASIS MATRICES!

- $_{\mathcal{W}}[L]_{\mathcal{V}} = [\mathcal{W}]^{-1} L [\mathcal{V}]$
  $= [\mathcal{W}]^{-1}\left([\mathcal{W}']\, _{\mathcal{W}'}[L]_{\mathcal{V}'} [\mathcal{V}']^{-1}\right)[\mathcal{V}]$
  $= \left([\mathcal{W}]^{-1}[\mathcal{W}']\right) {_{\mathcal{W}'}[L]_{\mathcal{V}'}} \left([\mathcal{V}']^{-1}[\mathcal{V}]\right)$

(b) A CHANGE-OF BASIS MATRIX IS A SQUARE, INVERTIBLE MATRIX GIVING THE ISOMORPHISM OF $\mathbb{R}^k$ THAT MATCHES $\mathcal{B}$-COORDINATES OF A VECTOR WITH ITS $\mathcal{B}'$-COORDINATES IN SOME VECTOR SPACE U.

$$\underbrace{[\mathcal{B}']^{-1}}_{\text{coords}}\underbrace{[\mathcal{B}]}_{\text{l.c.}}$$

$\mathcal{B}'$-COORD.'S IN $\mathbb{R}^k$ ← VECTOR IN U ← $\mathcal{B}$-COORD'S IN $\mathbb{R}^k$

WE CAN COMPUTE SUCH A MATRIX ONE COLUMN AT A TIME, AS USUAL:

j^{th} COLUMN OF $[\mathcal{B}']^{-1}[\mathcal{B}] = \left([\mathcal{B}']^{-1}[\mathcal{B}]\right)\vec{e}_j$
$= [\mathcal{B}']^{-1}\left([\mathcal{B}]\vec{e}_j\right)$
$= [\mathcal{B}']^{-1}\left(j^{\text{th}} \text{ BASIS VECTOR IN } \mathcal{B}\right)$
$= \mathcal{B}'$- COORDINATES OF THE j^{th} VECTOR IN $\mathcal{B}$.

(c) THE CHANGE-OF-BASIS MATRIX $[\mathcal{B}']^{-1}[\mathcal{B}]$ ($\mathcal{B}$-COORD'S $\mapsto$ $\mathcal{B}'$-COORD.'S)

AND ITS OPPOSITE, $[\mathcal{B}]^{-1}[\mathcal{B}']$ ($\mathcal{B}'$-COORD.'S $\mapsto$ $\mathcal{B}$-COORD'S)

ARE INVERSES:

$$\left([\mathcal{B}']^{-1}[\mathcal{B}]\right)\left([\mathcal{B}]^{-1}[\mathcal{B}']\right) = [\mathcal{B}']^{-1}\left(\cancel{[\mathcal{B}][\mathcal{B}]^{-1}}\right)[\mathcal{B}'] = \cancel{[\mathcal{B}']^{-1}[\mathcal{B}']} = I$$

AND

$$\left([\mathcal{B}]^{-1}[\mathcal{B}']\right)\left([\mathcal{B}']^{-1}[\mathcal{B}]\right) = [\mathcal{B}]^{-1}\left(\cancel{[\mathcal{B}'][\mathcal{B}']^{-1}}\right)[\mathcal{B}] = \cancel{[\mathcal{B}]^{-1}[\mathcal{B}]} = I$$

(d) CHANGE-OF-BASIS MATRICES FOR $\mathbb{R}^n$ CAN BE COMPUTED VIA MATRIX-REDUCTION! (WHAT ELSE? ☺)

TO FIND THE $\mathcal{B}'$- COORDINATES OF ANY VECTOR $\vec{x} \in \mathbb{R}^n$, WE'D SOLVE $[\mathcal{B}' \mid \vec{x}]$. SO, TO FIND THE $\mathcal{B}'$ COORDINATES OF ALL OF THE VECTORS OF $\mathcal{B}$, WE JUST MULTIPLY-AUGMENT, FORMING $[\mathcal{B}' \mid \mathcal{B}]$ — THIS REDUCES TO $[I \mid ✱]$

↳ CHANGE-OF-BASIS MATRIX!

✱ VIA THE F.T.V.S, WE CAN USE THIS METHOD FOR ANY OTHER FINITE-DIMENSIONAL VECTOR SPACE, TOO!

(e) A CHANGE-OF-BASIS MATRIX $[\mathcal{B}']^{-1}[\mathcal{B}] = [\mathcal{B}']^{-1} \circ id \circ [\mathcal{B}] = {_{\mathcal{B}'}[id]_{\mathcal{B}}}$

— I.E., IT HAS ALL ELEMENTS OF A MATRIX FOR A L.T. EXCEPT FOR THE L.T. ITSELF: IT TAKES A L.C. OF $\mathcal{B}$, DOES NOTHING TO IT, THEN TAKES $\mathcal{B}'$-COORDINATES OF THE RESULT.

2. (a) A LINEAR TRANSFORMATION $L: V \to W$ CAN BE REPRESENTED BY MANY MATRICES, DEPENDING UPON THE BASES CHOSEN FOR V AND W — SO, THE QUESTION THAT RESULTS IS WHETHER TWO GIVEN MATRICES DO OR DO NOT ARISE AS MATRICES FOR THE SAME L.T. (POSSIBLY WITH RESPECT TO DIFFERENT BASES). IN LIGHT OF CHANGE OF BASIS, THIS AMOUNTS TO ASKING WHETHER TWO $m \times n$ MATRICES A AND B ARE RELATED BY $B = YAX$, WHERE X AND Y ARE INVERTIBLE MATRICES (OF SIZE $n \times n$ AND $m \times m$, RESPECTIVELY).

(b) IF $L: V \to W$, WE CAN CHOOSE BASES (SIMILAR TO THOSE IN THE RANK+NULLITY PROOF), AS FOLLOWS:

$$\mathcal{V} = (\vec{v}_1, \dots, \vec{v}_r ; \underbrace{\vec{k}_1, \dots, \vec{k}_n}_{\text{BASIS FOR KER } L}) \quad \text{FOR } V$$

$$\mathcal{W} = (L\vec{v}_1, \dots, L\vec{v}_r ; \underbrace{\vec{w}_1, \dots}_{\text{EXTEND } L\vec{v}_1, \dots, L\vec{v}_r \text{ TO A BASIS}}) \quad \text{FOR } W$$

THEN WE CAN EASILY COMPUTE ${}_{\mathcal{W}}[L]_{\mathcal{V}}$ TO BE AS FOLLOWS:

$$\left[\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right], \quad \text{WHERE } \begin{cases} r = \text{RANK } L \\ \#\text{COLS} = \text{DIM } V \\ \#\text{ROWS} = \text{DIM } W \end{cases}$$

THUS ANY OTHER $m \times n$ MATRIX FOR L IS EQUIVALENT TO THIS MATRIX — SO TO CHECK WHETHER TWO $m \times n$ MATRICES ARE EQUIVALENT, ALL WE HAVE TO CHECK IS WHETHER THEY HAVE THE SAME RANK.

3. $\mathcal{V} = (\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$; $\mathcal{V}' = (-\vec{v}_3 + \vec{v}_4, \vec{v}_3 - 3\vec{v}_4, 2\vec{v}_1 + \vec{v}_2, \vec{v}_1 - \vec{v}_2)$

$\mathcal{W} = (\vec{w}_1, \vec{w}_2, \vec{w}_3)$; $\mathcal{W}' = (3\vec{w}_1 + 2\vec{w}_2 - \vec{w}_3, \vec{w}_1 + \vec{w}_2 - \vec{w}_3, -2\vec{w}_1 + 3\vec{w}_2)$

* TRANSLATE THESE INTO COLUMN VECTORS VIA $\mathcal{V}$ + $\mathcal{W}$:

$$\mathcal{V} \leftrightarrow \left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \right); \quad \mathcal{V}' \leftrightarrow \left(\begin{bmatrix}0\\0\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\-3\end{bmatrix}, \begin{bmatrix}2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \right)$$

$$\mathcal{W} \leftrightarrow \left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right); \quad \mathcal{W}' \leftrightarrow \left(\begin{bmatrix}3\\2\\-1\end{bmatrix}, \begin{bmatrix}1\\1\\-1\end{bmatrix}, \begin{bmatrix}-2\\3\\0\end{bmatrix} \right)$$

(a) $$[\mathcal{V} \mid \mathcal{V}'] \rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -3 & 0 & 0 \end{array}\right]$$

ALREADY REDUCED!

$\therefore$ THIS GIVES THE CHANGE OF BASIS

SO $$[\mathcal{V}]^{-1}[\mathcal{V}'] = \begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 \end{bmatrix}$$

(b) $$[\mathcal{V}' \mid \mathcal{V}] \rightsquigarrow \left[\begin{array}{cccc|cccc} 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & -3/2 & -1/2 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & 0 & 1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/3 & -2/3 & 0 & 0 \end{array}\right]$$

CHANGE-OF-BASIS MATRIX

SO $$[\mathcal{V}']^{-1}[\mathcal{V}] = \begin{bmatrix} 0 & 0 & -3/2 & -1/2 \\ 0 & 0 & -1/2 & -1/2 \\ 1/3 & 1/3 & 0 & 0 \\ 1/3 & -2/3 & 0 & 0 \end{bmatrix}$$

(c) $$[\mathcal{W} \mid \mathcal{W}'] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & -2 \\ 0 & 1 & 0 & 2 & 1 & 3 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array}\right] \quad \text{SO} \quad [\mathcal{W}]^{-1}[\mathcal{W}'] = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & 3 \\ -1 & -1 & 0 \end{bmatrix}$$

CHANGE-OF-BASIS MATRIX

(d) $$[\mathcal{W}' \mid \mathcal{W}] \rightsquigarrow \left[\begin{array}{ccc|ccc} 3 & 1 & -2 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/8 & 1/4 & 5/8 \\ 0 & 1 & 0 & -3/8 & -1/4 & -13/8 \\ 0 & 0 & 1 & -1/8 & 1/4 & 1/8 \end{array}\right]$$

CHANGE-OF-BASIS MATRIX

SO $$[\mathcal{W}']^{-1}[\mathcal{W}] = \begin{bmatrix} 3/8 & 1/4 & 5/8 \\ -3/8 & -1/4 & -13/8 \\ -1/8 & 1/4 & 1/8 \end{bmatrix}$$

4. (a) IF ${}_{\mathcal{W}}[L]_{\mathcal{V}} = \begin{bmatrix} -2 & -2 & -29 & -11 \\ 3 & 3 & -20 & -8 \\ 0 & 0 & 11 & 5 \end{bmatrix}$, THEN TO FIND ${}_{\mathcal{W}'}[L]_{\mathcal{V}'}$, WE JUST NEED TO COMPOSE WITH THE CORRECT CHANGE-OF-BASIS MATRICES:

- ON THE LEFT, WITH $[\mathcal{W}']^{-1}[\mathcal{W}]$
  (TO SWITCH FROM $\mathcal{W}$-COORD.'S TO $\mathcal{W}'$ COORDS IN W)
- ON THE RIGHT, WITH $[\mathcal{V}]^{-1}[\mathcal{V}']$
  (TO TAKE $\mathcal{V}'$ COORD.'S + CHANGE THEM TO $\mathcal{V}$-COORD'S IN V)

$$
{}_{\mathcal{W}'}[L]_{\mathcal{V}'} = \begin{bmatrix} 3/8 & 1/4 & 5/8 \\ -3/8 & -1/4 & -13/8 \\ -1/8 & 1/4 & 1/8 \end{bmatrix} \underbrace{\begin{bmatrix} -2 & -2 & -29 & -11 \\ 3 & 3 & -20 & -8 \\ 0 & 0 & 11 & 5 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 \end{bmatrix}}
$$

$$
= \begin{bmatrix} 3/8 & 1/4 & 5/8 \\ -3/8 & -1/4 & -13/8 \\ -1/8 & 1/4 & 1/8 \end{bmatrix} \begin{bmatrix} 29-11 & -29+33 & -4-2 & 0-0 \\ 20-8 & -20+24 & 6+3 & 0-0 \\ -11+5 & 11-15 & 0+0 & 0-0 \end{bmatrix}
$$

$$
= \underbrace{\begin{bmatrix} 3/8 & 1/4 & 5/8 \\ -3/8 & -1/4 & -13/8 \\ -1/8 & 1/4 & 1/8 \end{bmatrix} \begin{bmatrix} 18 & 4 & -6 & 0 \\ 12 & 4 & 9 & 0 \\ -6 & -4 & 0 & 0 \end{bmatrix}}
$$

$$
= \begin{bmatrix} 27/4 + 3 - 15/4 & 3/2 + 1 - 5/2 & -9/4 + 9/4 + 0 & 0+0+0 \\ -27/4 - 3 + 39/4 & -3/2 - 1 + 13/2 & 9/4 - 9/4 + 0 & 0+0+0 \\ -9/4 + 3 - 3/4 & -1/2 + 1 - 1/2 & 3/4 + 9/4 + 0 & 0+0+0 \end{bmatrix}
$$

$$
= \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}
$$

← REMEMBER THAT THE j^{TH} COLUMN OF THIS MATRIX GIVES THE $\mathcal{W}'$-COORDINATES OF THE j^{TH} BASIS VECTOR OF $\mathcal{V}'$!

(b) L SENDS THE 1^{ST} VECTOR OF $\mathcal{V}'$ TO 6 TIMES THE 1^{ST} VECTOR OF $\mathcal{W}'$; IT SENDS THE 2^{ND} VECTOR OF $\mathcal{V}'$ TO 4 TIMES THE 2^{ND} VECTOR OF $\mathcal{W}'$; IT SENDS THE 3^{RD} VECTOR OF $\mathcal{V}'$ TO 3 TIMES THE 3^{RD} VECTOR OF $\mathcal{W}'$; AND IT SENDS THE 4^{TH} VECTOR OF $\mathcal{V}'$ TO THE ZERO VECTOR.

(c) THE ABOVE COMPUTATION DEMONSTRATES THE UTILITY OF CHANGE OF BASIS: IT ALLOWS US TO SIMPLIFY OUR PRESENTATION OF A LINEAR TRANSFORMATION INTO A FORM (WITH LOTS OF 0'S) WHOSE MEANING CAN BE VERY QUICKLY UNDERSTOOD.

(* IF WE'RE ABLE TO FIND "GOOD" BASES!)

5. (AAH... COLUMN VECTORS AND NICE SIMPLE BASES... TIME TO UNWIND FROM THE COMPUTATIONAL MESS ABOVE!)

(a)
$$
\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\alpha & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ +\alpha \end{array} \rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]
$$

NEW BASIS — STANDARD BASIS — CHANGE-OF-BASIS MATRIX

(b)
$$
\left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \text{SWAP} \rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]
$$

NEW BASIS — STANDARD BASIS — CHANGE-OF-BASIS MATRIX

(c)
$$
\cdot\alpha \rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/\alpha & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]
$$

NEW BASIS — STANDARD BASIS — CHANGE-OF-BASIS MATRIX

6. (i) (a) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1+\alpha d_1 & b_2+\alpha d_2 & b_3+\alpha d_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix}$$

ROW OPERATION: ADD α TIMES THE 4TH ROW TO THE 2ND ROW!

(b) $$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix}$$

ROW OPERATION: EXCHANGE THE 1ST + 2ND ROWS!

(c) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ \alpha b_1 & \alpha b_2 & \alpha b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{bmatrix}$$

ROW OPERATION: SCALE THE 2ND ROW BY A FACTOR OF α!

(ii) WHAT WE'VE SEEN ABOVE IS THAT ALL THREE OF OUR BASIC MATRIX-REDUCTION OPERATIONS CAN BE VIEWED AS SIMPLY CHANGING BASIS FOR THE CODOMAIN — SO, WHEN WE REDUCE A MATRIX TO SOLVE A LINEAR SYSTEM (I.E., A LINEAR COMBINATION PROBLEM), WHAT WE'RE DOING IS CHANGING THE BASIS WE USE TO EXPRESS IT, UNTIL THE NEW BUILDING BLOCKS ARE SIMPLE AND IT'S EASY TO TELL HOW TO BUILD THE TARGET!!