Linear Systems

- Definitions:
    - *linear system*; *standard form*
    - *homogeneous* vs. *inhomogeneous* system   (be sure to put equations in standard form first!)
    - *consistent* vs. *inconsistent* system   (at least one solution vs. no solutions)
    - *implicit* vs. *explicit* (or *parametric*) presentation of a system's solutions

- Solving:
    - choose pivots, eliminate variables, check for consistency
    - if consistent, locate the free variables and solve for the pivots

Augmented matrices

- Augmented matrix notation   (put equations in standard form first!)
    - rows $\leftrightarrow$ equations
    - pre-final columns $\leftrightarrow$ variables, storing their coefficients
    - final column stores the constant terms

- Basic row operations:
    - exchange two rows
    - scale a row by a nonzero constant
    - add a multiple of one row to another

- Definitions:
    - *pivot*   (first nonzero entry in a row, if any, left of the line)
    - *pivot variable* vs. *free variable*

- Pivoting and reduction of augmented matrices (*Gaussian elimination*)
    - work left-to-right through the columns, looking for new pivots
    - when a new pivot is found:
        - · exchange rows (if necessary),
        - · scale the row to make the pivot 1, and
        - · add appropriate multiples of the row to kill all other nonzero entries in the column
    - finished when the pivots proceed downward and rightward, pivots 1 with all entries above and below them 0

- Solving systems (after fully reducing the augmented matrix)
    - if any non-pivot row has a nonzero entry to the right of the line, the system is **inconsistent** (the end!)
    - *otherwise*, the system is **consistent**; locate the free variables (if any) and solve for the pivots

Logical relationships

To think through the possibilities for a system:

- Suppose that the augmented matrix has been fully reduced
    - assume that the pivots (if any) run diagonally down & right from the top-left entry
    - every variable is either a pivot variable or a free variable (so pivots + free = total # of variables)

- Could there be a non-pivot row with a nonzero entry to the right of the line?
    - a non-pivot row (all 0's to the left of the line) with nonzero entry to the right of the line $\Rightarrow$ the system is **inconsistent**
    - no non-pivot rows with nonzero entries to the right of the line $\Rightarrow$ the system is **consistent**
        - · in particular, the system must be consistent if every row has a pivot or if the system is homogeneous

- If consistent, how many free variables could there be?
    - no free variables $\Rightarrow$ **unique solution**
    - at least one free variable $\Rightarrow$ **more than one solution** (infinitely many, for real numbers)